Unknown Thru Calibration Algorithm

Short-Open-Load-Reciprocal (SOLR)

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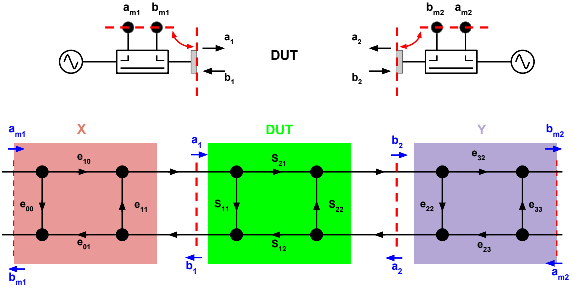
1. Calibration Kit
   - SHORT, OPEN and LOAD standards
   - THRU standard and the need for SOLR calibration
2. SOL (1-Port)
   - Error models
   - Error terms calculation
3. SOLT (2-Port)
   - from $[S]$ measurements
   - from waves measurements
4. SOLR (2-Port)
   - from $[S]$ measurements
   - from waves measurements

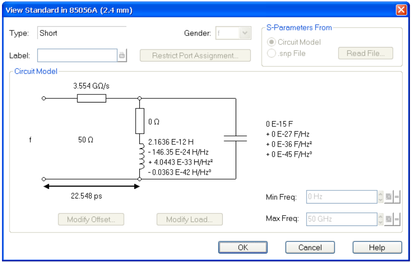
NVNA architecture and 8-terms error model for calibration

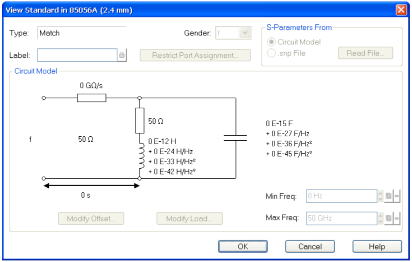


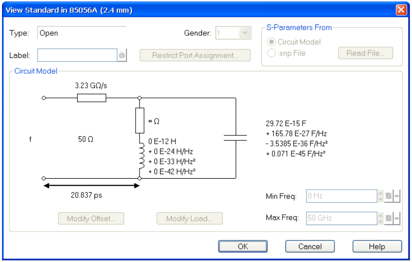
During SOLx calibration, Γ_{Short}, Γ_{Open}, and Γ_{Load} are assumed to be totally known.

- SOLT : $[S_{Thru}]$ is assumed to be totally known. Model of the 'Thru' may be not accurate enough.
- SOLR : $[S_{Thru}]$ is unknown but RECIPROCAL (valid for passive device). $[S_{Thru}]$ values are identified during the calibration.

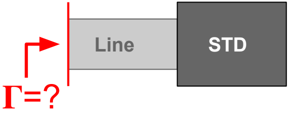
Models parameters for standards







Open: C0, C1, C2 and C3
Short: L0, L1, L2 and L3
Load: Z0;
Line: Delay, Loss and Z0.



SHORT standard calculation

- Inductance frequency polynomial model

$$L(f) = L_0 + L_1.f + L_2.f^2 + L_3.f^3$$

$$\Gamma_L = \frac{j.\omega.L(f) - Z_0}{j.\omega.L(f) + Z_0}$$

- Offset length model

$$T_{Loss} = e^{-\frac{\text{Delay}}{Z_0}.\text{Loss}.\sqrt{f_{(GHz)}}}$$

$$T_{Delay} = e^{-j.4\pi.f.\text{Delay}}$$

- S_{11} of the standard

$$\Gamma_{Short} = \Gamma_L.T_{Loss}.T_{Delay}$$

OPEN standard calculation

- Capacitance frequency polynomial model

$$C(f) = C_0 + C_1.f + C_2.f^2 + C_3.f^3$$

$$\Gamma_C = \frac{1 - j.\omega.Z_0.C(f)}{1 - j.\omega.Z_0.C(f)}$$

- Offset length model

$$T_{Loss} = e^{-\frac{\text{Delay}}{Z_0}.\text{Loss}.\sqrt{f_{(GHz)}}}$$

$$T_{Delay} = e^{-j.4\pi.f.\text{Delay}}$$

- S_{11} of the standard

$$\Gamma_{Open} = \Gamma_C.T_{Loss}.T_{Delay}$$

LOAD standard calculation

- Fixed impedance

$$Z = R + j.L_T.\omega$$

$$\Gamma_Z = \frac{Z - Z_0}{Z + Z_0}$$

- Offset length model

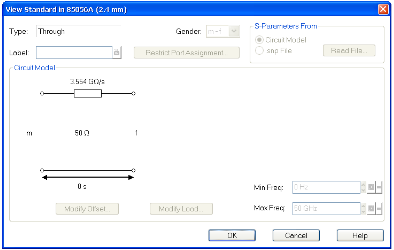
$$T_{Loss} = e^{-\frac{\text{Delay}}{Z_0}.\text{Loss}.\sqrt{f_{(GHz)}}}$$

$$T_{Delay} = e^{-j.4\pi.f.\text{Delay}}$$

- S_{11} of the standard

$$\Gamma_{Load} = \Gamma_Z.T_{Loss}.T_{Delay}$$

THRU standard calculation



- Ideal Thru

$$[S] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Modeled Thru

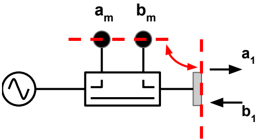
$$[S] = \begin{bmatrix} 0 & e^{-\gamma.L} \\ e^{-\gamma.L} & 0 \end{bmatrix}$$

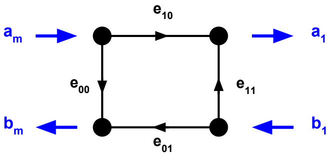
with

$$\gamma.L = \frac{\tau}{2.Z_0}.\text{Loss}.\sqrt{f_{(GHz)}} + j.2\pi.f.\tau$$

- Issues
  - τ may be unknown
  - Loss may be unknown
  - Are S_{11} and S_{22} equal to zero ?
  - Validity of the model ?

One port relative calibration error models





LSNA papers

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

VNA papers

$$\begin{pmatrix} b_{m1} \\ a_1 \end{pmatrix} = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} \frac{e_{10}e_{01}-e_{11}e_{00}}{e_{01}} & \frac{e_{11}}{e_{01}} \\ -\frac{e_{00}}{e_{01}} & \frac{1}{e_{01}} \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$$\begin{pmatrix} b_{m1} \\ a_1 \end{pmatrix} = \begin{bmatrix} -\frac{\gamma}{\delta} & \frac{1}{\delta} \\ \frac{\alpha\delta-\beta\gamma}{\delta} & \frac{\beta}{\delta} \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_1 \end{pmatrix}$$

One port relative calibration

$$\Gamma_m = \frac{b_{m1}}{a_{m1}} \quad \text{and} \quad \Gamma = \frac{b_1}{a_1}$$

LSNA papers

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

VNA papers

$$\begin{pmatrix} b_{m1} \\ a_1 \end{pmatrix} = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \alpha_1 . \begin{bmatrix} 1 & \beta_1' \\ \gamma_1' & \delta_1' \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$$\Gamma = \frac{\gamma_1' + \delta_1'.\Gamma_m}{1 + \beta_1'.\Gamma_m}$$

$$\Gamma = \beta_1'.\Gamma.\Gamma_m + \gamma_1' + \delta_1'.\Gamma_m$$

$$\Gamma = \frac{\Gamma_m - e_{00}}{\Gamma_m.e_{11} - \Delta e}$$

with $\Delta e = e_{00}e_{11} - e_{10}e_{01}$

$$e_{00} + \Gamma.\Gamma_m.e_{11} - \Gamma.\Delta e = \Gamma_m$$

SOL : SHORT-OPEN-LOAD

$$\Gamma_{<std>} = \frac{b_1}{a_1} \quad \text{and} \quad \Gamma_{m<std>} = \frac{b_{m1}}{a_{m1}} \quad \text{with} \quad <std>= \text{Short ; Open ; Load}$$

LSNA papers

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \alpha_1 . \begin{bmatrix} 1 & \beta_1' \\ \gamma_1' & \delta_1' \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$$\Gamma_{<std>} = \beta' . \Gamma_{<std>} . \Gamma_{m<std>} + \gamma' + \delta' . \Gamma_{m<std>}$$

$$\begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix} = \begin{pmatrix} \beta_1' \\ \gamma_1' \\ \delta_1' \end{pmatrix} . \begin{bmatrix} -\Gamma_{mS}.\Gamma_S & 1 & \Gamma_{mS} \\ -\Gamma_{mO}.\Gamma_O & 1 & \Gamma_{mO} \\ -\Gamma_{mL}.\Gamma_L & 1 & \Gamma_{mL} \end{bmatrix}$$

$$\begin{bmatrix} -\Gamma_{mS}.\Gamma_S & 1 & \Gamma_{mS} \\ -\Gamma_{mO}.\Gamma_O & 1 & \Gamma_{mO} \\ -\Gamma_{mL}.\Gamma_L & 1 & \Gamma_{mL} \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix} = \begin{pmatrix} \beta_1' \\ \gamma_1' \\ \delta_1' \end{pmatrix}$$

VNA papers

$$\begin{pmatrix} b_{m1} \\ a_1 \end{pmatrix} = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_1 \end{pmatrix}$$

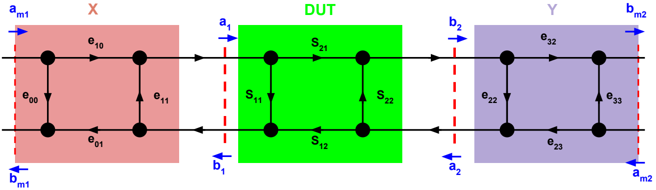
$$e_{00} + \Gamma_{<std>} . \Gamma_{m<std>} . e_{11} - \Gamma_{<std>} . \Delta e = \Gamma_{m<std>}$$

$$\begin{pmatrix} \Gamma_{mS} \\ \Gamma_{mO} \\ \Gamma_{mL} \end{pmatrix} = \begin{pmatrix} e_{00} \\ e_{11} \\ \Delta e \end{pmatrix} . \begin{bmatrix} 1 & -\Gamma_{mS}.\Gamma_S & -\Gamma_S \\ 1 & -\Gamma_{mO}.\Gamma_O & -\Gamma_O \\ 1 & -\Gamma_{mL}.\Gamma_L & -\Gamma_L \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\Gamma_{mS}.\Gamma_S & -\Gamma_S \\ 1 & -\Gamma_{mO}.\Gamma_O & -\Gamma_O \\ 1 & -\Gamma_{mL}.\Gamma_L & -\Gamma_L \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_{mS} \\ \Gamma_{mO} \\ \Gamma_{mL} \end{pmatrix} = \begin{pmatrix} e_{00} \\ e_{11} \\ \Delta e \end{pmatrix}$$

SOLT : general concepts

- Flow Graph



- $[T]$ matrix definition

$$[T] = \frac{1}{S_{21}} . \begin{bmatrix} 1 & -S_{22} \\ S_{11} & S_{12}.S_{21} - S_{11}.S_{22} \end{bmatrix} \Leftrightarrow [S] = \frac{1}{T_{11}} . \begin{bmatrix} T_{21} & T_{11}.T_{22} - T_{12}.T_{21} \\ 1 & -T_{12} \end{bmatrix}$$

- Cascading and de-embedding properties

$$[T_m] = [T_X].[T_{DUT}].[T_Y] \quad \Leftrightarrow \quad [T_{DUT}] = [T_X]^{-1}.[T_m].[T_Y]^{-1}$$

SOLT from $[S]$ measurements : 7 error terms to identify

- Port 1 *(Fwd)*

$$\begin{pmatrix} b_{m1} \\ a_1 \end{pmatrix} = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_1 \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 1 $\rightarrow e_{00}$, e_{11}, and $e_{10}.e_{01} = \Delta e_X - e_{00}.e_{11}$

- Port 2 *(Rev)*

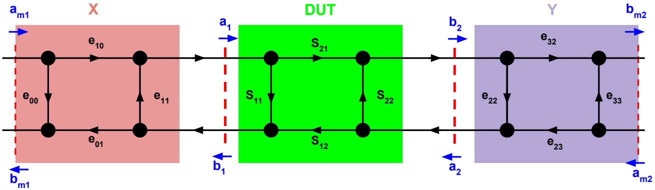
$$\begin{pmatrix} b_{m2} \\ a_2 \end{pmatrix} = \begin{bmatrix} e_{33} & e_{32} \\ e_{23} & e_{22} \end{bmatrix} . \begin{pmatrix} a_{m2} \\ b_2 \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 2 $\rightarrow e_{22}$, e_{33}, and $e_{32}.e_{23} = \Delta e_Y - e_{22}.e_{33}$

- Transfert

$\Rightarrow [S_{Thru}]$ *(Fwd and Rev)* $\rightarrow e_{10}.e_{32}$
we can use 1 (direct solution) up to 4 equations (least-square method)

SOLT : THRU calibration $(e_{10}.e_{32})$ and $[S]$ measurements



- Finding $(e_{10}.e_{32})$ from $[S_{THRU}]$

$$[T_m] = [T_X].[T_{THRU}].[T_Y]$$

$$[T_m] = \frac{1}{e_{10}.e_{32}}.\begin{bmatrix} 1 & -e_{11} \\ e_{00} & -\Delta e_X \end{bmatrix}.[T_{THRU}].\begin{bmatrix} 1 & -e_{33} \\ e_{22} & -\Delta e_Y \end{bmatrix}$$

- Calibrated $[S_{DUT}]$ measurements

$$[T_{DUT}] = (e_{10}.e_{32}).\begin{bmatrix} 1 & -e_{11} \\ e_{00} & -\Delta e_X \end{bmatrix}^{-1}.[T_m].\begin{bmatrix} 1 & -e_{33} \\ e_{22} & -\Delta e_Y \end{bmatrix}^{-1}$$

with $\Delta e_X = e_{00}e_{11} - e_{10}e_{01}$ and $\Delta e_Y = e_{22}e_{33} - e_{32}e_{23}$

SOLT from waves : 7 error terms to identify

- Port 1 *(Fwd)*

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 1 $\rightarrow \beta_1, \gamma_1, \delta_1$

- Port 2 *(Rev)*

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \alpha_2 . \begin{bmatrix} 1 & \beta_2' \\ \gamma_2' & \delta_2' \end{bmatrix} . \begin{pmatrix} a_{m2} \\ b_{m2} \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 2 $\rightarrow \beta_2', \gamma_2', \delta_2'$

- Transfert

$\Rightarrow [S_{Thru}]$ *(Fwd and Rev)* $\rightarrow \alpha_2$
we can use 1 (direct solution) up to 4 equations (least-square method)

SOLT : THRU (transfert) $\Rightarrow \alpha_2$

- Calibration error matrices

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} . \begin{pmatrix} a_{1m} \\ b_{1m} \end{pmatrix} \text{ and } \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \alpha_2 . \begin{bmatrix} 1 & \beta_2' \\ \gamma_2' & \delta_2' \end{bmatrix} . \begin{pmatrix} a_{2m} \\ b_{2m} \end{pmatrix}$$

- α_2 from $b_2 = S_{21}.a_1 + S_{22}.a_2$ in forward mode

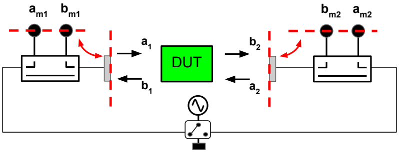
$$\alpha_2 = \frac{S_{21}.(a_{m1} + \beta_1.b_{m1})}{\gamma_2'.a_{m2} + \delta_2'.b_{m2} - S_{22}.(a_{m2} + \beta_2'.b_{m2})}$$

- With an ideal THRU

$$[S_{THRU}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\alpha_2 = \frac{a_{m1} + \beta_1.b_{m1}}{\gamma_2'.a_{m2} + \delta_2'.b_{m2}}$$

SOLT : $[S]$ from calibrated waves measurements



Two independent measurements :

- Forward mode
- Reverse mode

$$\begin{bmatrix} b_1^{Fwd} & b_1^{Rev} \\ b_2^{Fwd} & b_2^{Rev} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1^{Fwd} & a_1^{Rev} \\ a_2^{Fwd} & a_2^{Rev} \end{bmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} b_1^{Fwd} & b_1^{Rev} \\ b_2^{Fwd} & b_2^{Rev} \end{bmatrix} \cdot \begin{bmatrix} a_1^{Fwd} & a_1^{Rev} \\ a_2^{Fwd} & a_2^{Rev} \end{bmatrix}^{-1}$$

SOLR : SHORT-OPEN-LOAD-RECIPROCAL

Andrea Ferrero

IEEE MICROWAVE AND GUIDED WAVE LETTERS, VOL. 2, NO. 12, DECEMBER 1992 505

Two-Port Network Analyzer Calibration Using an Unknown "*Thru*"

Andrea Ferrero, *Member, IEEE*, and Umberto Pisani

***Abstract*—A procedure performed by using a generic two port reciprocal network instead of a standard *thru* in a full two-port error correction of an automatic network analyzer is presented. Although it can be applied to any type of waveguide system the proposed technique is particularly useful with noninsertable coaxial or on-wafer devices. Experimental comparisons show that the suggested procedure provides a great degree of accuracy.**

I. Introduction

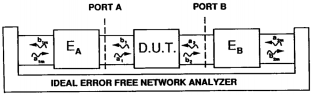
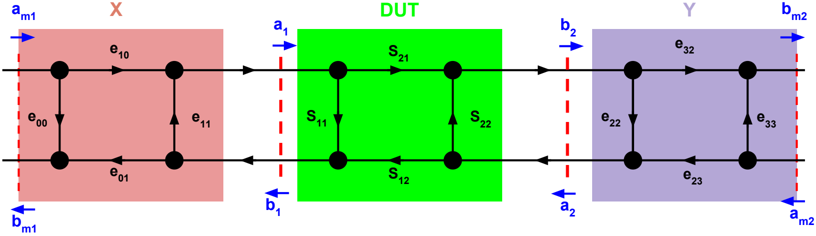


Fig. 1. Error box NWA model.

A. Ferrero, U. Pisani,
"Two-port network analyzer calibration using an unknown 'thru'"
IEEE Microwave and Guided Wave Letters, Vol. 2, No. 12, 1992, pp. 505-507

SOLR : General Concepts

- Flow Graph



- $[T]$ matrix definition

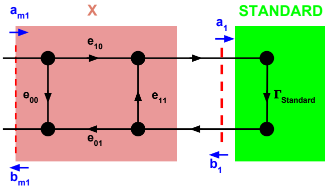
$$[T] = \frac{1}{S_{21}} . \begin{bmatrix} 1 & -S_{22} \\ S_{11} & S_{12}.S_{21} - S_{11}.S_{22} \end{bmatrix}$$

- Reciprocity assumption

$$S_{12} = S_{21} \quad \Rightarrow \quad \det([T]) = 1$$

SOLR : Short-Open-Load calibration on port 1 (forward)

$< std >$=Short ; Open ; Load



$$\Gamma_{<std>} = \frac{b_1}{a_1} \quad \text{and} \quad \Gamma_{m<std>} = \frac{b_{m1}}{a_{m1}}$$

$$\begin{bmatrix} 1 & -\Gamma_{mS}.\Gamma_S & -\Gamma_S \\ 1 & -\Gamma_{mO}.\Gamma_O & -\Gamma_O \\ 1 & -\Gamma_{mL}.\Gamma_L & -\Gamma_L \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_{mS} \\ \Gamma_{mO} \\ \Gamma_{mL} \end{pmatrix} = \begin{pmatrix} e_{00} \\ e_{11} \\ \Delta X \end{pmatrix}$$

with $\Delta X = e_{00}.e_{11} - e_{10}.e_{01}$

$\Downarrow$

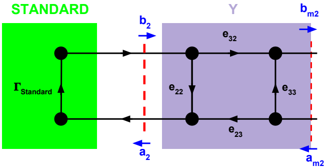
e_{00}, e_{11}, and $e_{10}.e_{01}$ are known

SOLR : Short-Open-Load calibration on port 2 (reverse)

$< std >$=Short ; Open ; Load

$$\Gamma_{<std>} = \frac{b_2}{a_2} \quad \text{and} \quad \Gamma_{m<std>} = \frac{b_{m2}}{a_{m2}}$$

$$\begin{bmatrix} 1 & -\Gamma_{mS}.\Gamma_S & -\Gamma_S \\ 1 & -\Gamma_{mO}.\Gamma_O & -\Gamma_O \\ 1 & -\Gamma_{mL}.\Gamma_L & -\Gamma_L \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_{mS} \\ \Gamma_{mO} \\ \Gamma_{mL} \end{pmatrix} = \begin{pmatrix} e_{33} \\ e_{22} \\ \Delta Y \end{pmatrix}$$

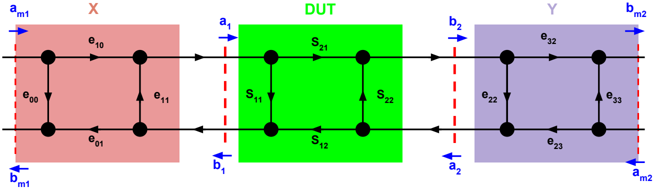


with $\Delta Y = e_{33}.e_{22} - e_{32}.e_{23}$

$\Downarrow$

e_{33}, e_{22}, and $e_{32}.e_{23}$ are known

SOLR : Reciprocity on the 'Unknown Thru'



$$[T_{DUT}] = (e_{10}e_{32}).\begin{bmatrix} 1 & -e_{11} \\ e_{00} & -\Delta X \end{bmatrix}^{-1}.[T_m].\begin{bmatrix} 1 & -e_{33} \\ e_{22} & -\Delta Y \end{bmatrix}^{-1}$$

with $\Delta X = e_{00}e_{11} - e_{10}e_{01}$ and $\Delta Y = e_{33}e_{22} - e_{32}e_{23}$

- Reciprocity

$$\det([T_{DUT}]) = 1 \quad \Rightarrow \quad (e_{10}e_{32})^2 = \frac{(e_{10}e_{01}).(e_{32}e_{23})}{\det([T_m])}$$

$$(e_{10}e_{32}) = \pm\sqrt{\frac{(e_{10}e_{01}).(e_{32}e_{23})}{\det([T_m])}}$$

SOLR from waves : 7 error terms to identify

- Port 1 *(Fwd)*

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} . \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 1 $\rightarrow \beta_1, \gamma_1, \delta_1$

- Port 2 *(Rev)*

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \alpha_2 . \begin{bmatrix} 1 & \beta'_2 \\ \gamma'_2 & \delta'_2 \end{bmatrix} . \begin{pmatrix} a_{m2} \\ b_{m2} \end{pmatrix}$$

$\Rightarrow$ Short-Open-Load on port 2 $\rightarrow \beta'_2, \gamma'_2, \delta'_2$

- Transfert

$\Rightarrow$ Reciprocity *(Fwd and Rev)* $\rightarrow \alpha_2$

SOLR : SHORT-OPEN-LOAD $\Rightarrow \beta_1 ; \gamma_1 ; \delta_1 ; \beta'_2 ; \gamma'_2 ; \delta'_2$

On port i, for $< std >$=Short ; Open ; Load

$$\Gamma_{<std>} = \frac{b_i}{a_i} \quad \text{and} \quad \Gamma_{m<std>} = \frac{b_{mi}}{a_{mi}}$$

- Port 1

$$\begin{bmatrix} -\Gamma_{mS}.\Gamma_S & 1 & \Gamma_{mS} \\ -\Gamma_{mO}.\Gamma_O & 1 & \Gamma_{mO} \\ -\Gamma_{mL}.\Gamma_L & 1 & \Gamma_{mL} \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \gamma_1 \\ \delta_1 \end{pmatrix}$$

- Port 2

$$\begin{bmatrix} -\Gamma_{mS}.\Gamma_S & 1 & \Gamma_{mS} \\ -\Gamma_{mO}.\Gamma_O & 1 & \Gamma_{mO} \\ -\Gamma_{mL}.\Gamma_L & 1 & \Gamma_{mL} \end{bmatrix}^{-1} . \begin{pmatrix} \Gamma_S \\ \Gamma_O \\ \Gamma_L \end{pmatrix} = \begin{pmatrix} \beta'_2 \\ \gamma'_2 \\ \delta'_2 \end{pmatrix}$$

SOLR : Reciprocity (transfert) $\Rightarrow \alpha_2$

- Uncompleted Relative Calibration

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix} \text{ and } \begin{pmatrix} a'_2 \\ b'_2 \end{pmatrix} = \begin{bmatrix} 1 & \beta'_2 \\ \gamma'_2 & \delta'_2 \end{bmatrix} \cdot \begin{pmatrix} a_{m2} \\ b_{m2} \end{pmatrix}$$

- $[X]$ is the partially calibrated $[S_{thru}]$ measurement

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \begin{bmatrix} b_1^F & b_1^R \\ {b'}_2^F & {b'}_2^R \end{bmatrix} \cdot \begin{bmatrix} a_1^F & a_1^R \\ {a'}_2^F & {a'}_2^R \end{bmatrix}^{-1}$$

- Finding α_2 from reciprocity assumption $(S_{21} = S_{12})$

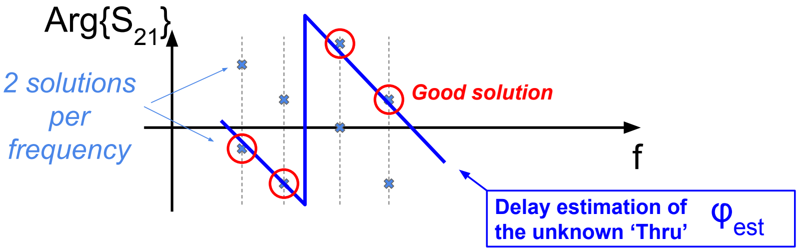
$$[S_{thru}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12}/\alpha_2 \\ X_{21}.\alpha_2 & X_{22} \end{bmatrix} \text{ then } \alpha_2 = \pm\sqrt{\frac{X_{12}}{X_{21}}}$$

SOLR : Root solution for α_2

$$\alpha_2 = \pm\sqrt{\frac{X_{12}}{X_{21}}}$$

$$S_{21} = X_{21}.\alpha_2$$

Pick up the solution close to $\phi_{est} = -2\pi.f.\tau$



SOLR : Complete relative calibration

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_{m1} \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \alpha_2 . \begin{bmatrix} 1 & \beta_2' \\ \gamma_2' & \delta_2' \end{bmatrix} \cdot \begin{pmatrix} a_{m2} \\ b_{m2} \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{pmatrix} = \begin{bmatrix} 1 & \beta_1 & 0 & 0 \\ \gamma_1 & \delta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & \beta_2 \\ 0 & 0 & \gamma_2 & \delta_2 \end{bmatrix} \cdot \begin{pmatrix} a_{m1} \\ b_{m1} \\ a_{m2} \\ b_{m2} \end{pmatrix}$$

Conclusion

- Some engineers should pay more attention on the validity of their 'Thru' CalKit model during VNA SOLT calibrations ;
- Using an 'Unknown Thru' calibration is a must compared to the SOLT method ;
- NVNA software developers should include SOLR method ;
- SOLR is easy to include in your code and obvious when the SOLT method already exists ;
- This presentation includes all you need to know for adding a SOLR calibration method on your NVNA system ;

Download this presentation

T. Reveyrand,
« *Unknown Thru Calibration Algorithm,* »
IEEE INMMiC 2018, Brive-la-Gaillarde, France, July 2018.

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