SOLT and SOLR calibration methods using a single multiport “thru” standard connection

Tibault Reveyrand, Silvia Hernández, Sébastien Mons, Edouard Ngoya
XLIM, 123 av. Albert Thomas, 87060 Limoges Cedex, France
tibo@xlim.fr

***Abstract*—This paper studies on the calibration of a multiport Nonlinear Vector Network Analyzer (NVNA), especially the relative calibration techniques. From the 8-term error model, we present two new alternative multiport “thru” calibrating procedures that reduce the number of connections to one by considering a multiport “thru” standard. Those two methods may be seen as multiport extensions of Short-Open-Load-Thru (SOLT) and Short-Open-Load-Reciprocal (SOLR) calibration techniques. Those new methods have been used successfully to calibrate a multiport NVNA. Finally multiport conversion from 8-term NVNA error model to standard 12-term VNA error model is presented.**

***Index Terms*—VNA, NVNA, Multiport Calibration, Error model, SOLT, SOLR**

I. INTRODUCTION

This paper proposes an extension of 8-term SOLT and SOLR relative calibration methods for multiport environments. The motivation for this work lies in the growing need for multiport coaxial NVNA measurements (MIMO systems, beamformers, multi-input PAs) and the desire to accelerate the calibration procedure.

The 8-term Short-Open-Load-Thru (SOLT) two-port VNA calibration method is based on two 1-port calibrations and the perfect knowledge of the [S] matrix of the “thru” standard (the “thru” can be considered ideal for insertable device measurements). The two-port VNA “unknown thru” or Short-Open-Load-Reciprocal (SOLR) calibration method has been proposed by A. Ferrero and U. Pisani in 1992 [1]. This calibration algorithm, assuming only the “thru” standard reciprocity, is particularly well suited for unknown “thru” standard model or non-insertable devices.

Multiport devices are often non-insertable devices. Commercial multiport VNA calibration propose the SOLR technique but this method, indeed, combine several 2-port SOLR calibration results. Thus, only the time delay of a 2-port “thru” is requested but a minimum of (n-1) transfert measurements and connexion/disconnection are needed [2], [3].

The idea of this paper is to consider a reciprocal multiport “thru” standard making possible to calibrate a multiport VNA with a minimum number of steps. Therefore, we will present a multiport SOLT method (for which the complete $[S_{Thru}]$ matrix of the multiport “thru” is known) and a multiport SOLR method (where the reciprocity assumption means we have an estimation of $\text{Arg}\{S_{Thru}(i,j)\}$ with $i \neq j$.

First, we will introduce the relative calibration formalism of an NVNA. In this formalism, error terms are not applied to S-parameters but waves. Thus, the NVNA 8-error terms model implicitly includes the switch-terms [4]. The SOLT and SOLR methods will be presented in the context of a 2-port NVNA. An extension to a multiport environment will be proposed. Those proposed methods are experimented with a Rohde & Schwarz ZVA 24 VNA. The new multiport SOLT and SOLR methods are applied with two types of multiport thru (with and without isolation). These calibrations are then compared to a conventional SOLR calibration (whose “thru” standard is a 2-port adapter used itaratively between each port).

Finally, the article will propose conversion formulas between the NVNA error terms and the 12-term model traditionally used in VNA firmwares.

II. RELATIVE CALIBRATION OF A NVNA

A. Calibration model

The 8-term model introduced with the LSNA [5], and valid for any VNA calibration [6], is used in this paper for NVNA relative calibration. Error term matrices with this model are represented with a cascade wave matrix (T-matrix) form.

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \begin{pmatrix} a_{1\,m} \\ b_{1\,m} \end{pmatrix} = \alpha_1 . \begin{bmatrix} 1 & \beta'_1 \\ \gamma'_1 & \delta'_1 \end{bmatrix} \begin{pmatrix} a_{1\,m} \\ b_{1\,m} \end{pmatrix} \quad (1)$$

Equation (1) illustrates the correction applied on the raw data $a_{1\,m}$ and $b_{1\,m}$ to obtain the calibrated waves a_1 and b_1 at the port 1 reference plane. β'_1, γ'_1 and δ'_1 are identified with a 1-port Short-Open-Load calibration at port 1. α_1 will be obtained from power and phase calibration of the NVNA at port 1 reference plane. Focusing on the relative calibration of a NVNA, we are considering $\alpha_1 = 1$ in this paper.

On every other NVNA port, notated i in equation (2), β'_i, γ'_i and δ'_i are deduced from a 1-port SOL applied on port i as will be explained in paragraph II-B.

$$\begin{pmatrix} a_i \\ b_i \end{pmatrix} = \begin{bmatrix} \alpha_i & \beta_i \\ \gamma_i & \delta_i \end{bmatrix} \begin{pmatrix} a_{i\,m} \\ b_{i\,m} \end{pmatrix} = \alpha_i . \begin{bmatrix} 1 & \beta'_i \\ \gamma'_i & \delta'_i \end{bmatrix} \begin{pmatrix} a_{i\,m} \\ b_{i\,m} \end{pmatrix} \quad (2)$$

α_i completes the abslotute calibration related to port i. This parameter, obtained from a “thru” standard characteristics, ensure a correct gain measurement between NVNA’s ports. Two algorithms dedicated to the calculus of α_i are presented in this paper: SOLT (in section III) and SOLR (in section IV) methods.

B. Calibration of a single port

The single port Short-Open-Load calibration method provide 3 error terms (β_i', γ_i' and δ_i') from calibrated and uncalibrated reflection coefficients values on port i.

$$\begin{pmatrix}\beta_i'\\ \gamma_i'\\ \delta_i'\end{pmatrix} = \begin{bmatrix}-\Gamma_{mS}.\Gamma_S & 1 & \Gamma_{mS}\\ -\Gamma_{mO}.\Gamma_O & 1 & \Gamma_{mO}\\ -\Gamma_{mL}.\Gamma_L & 1 & \Gamma_{mL}\end{bmatrix}^{-1} . \begin{pmatrix}\Gamma_S\\ \Gamma_O\\ \Gamma_L\end{pmatrix} \tag{3}$$

where, on port i, for $< std >$=Short ; Open ; Load

$$\Gamma_{<std>} = \frac{b_i}{a_i} \quad \text{and} \quad \Gamma_{m<std>} = \frac{b_{i\,m}}{a_{i\,m}}$$

Calibrated reflection coefficients Γ_{std} are given by the calibration kit manufacturer, and uncalibrated coefficients are the ratio of raw data waves.

C. Complete calibration

Each port of our VNA/NVNA is calibrated in Short-Open-Load according to equation (3). Our system is now partially calibrated.

$$\begin{pmatrix}\bar{a}_i\\ \bar{b}_i\end{pmatrix} = \begin{bmatrix}1 & \beta_i'\\ \gamma_i' & \delta_i'\end{bmatrix} . \begin{pmatrix}a_{i\,m}\\ b_{i\,m}\end{pmatrix} \tag{4}$$

The partially corrected waves are calculated from equation (4). Fully calibrated waves defined in equation (2) are related to partially calibrated waves presented in (4) as follow :

$$a_i = \alpha_i.\bar{a}_i \text{ and } b_i = \alpha_i.\bar{b}_i \tag{5}$$

We will present next how to extract α_i value on a 2-port and on a multiport VNA following the SOLT and SOLR methods with a single (eventually multiport) "thru" characterization.

III. SOLT METHOD

A. 2-port VNA

On a 2-port NVNA, the knowledge of the "thru" standard S-parameters makes possible to find out α_2 from a single forward measurement with the RF source active on port 1. According to the 1-port SOL calibration applied on both ports, half-calibrated waves $\bar{a}_1$, $\bar{b}_1$, $\bar{a}_2$ and $\bar{b}_2$ are given by equation (4). The normalization factor α_2, making port 2 calibration consistant with port 1 in magnitude and phase is then :

$$\alpha_2 = \left(\bar{b_2} - S_{22}.\bar{a}_2\right)^{-1}.\bar{a}_1.S_{21} \tag{6}$$

where S_{21} and S_{22} are the transmission and output reflection coefficient of the "thru" standard used in forward mode only.

B. n-port VNA

Expanding equation (6) in multiport leads us to :

$$\begin{pmatrix}\alpha_2\\ \vdots\\ \alpha_n\end{pmatrix} = \left[\text{diag}\begin{pmatrix}\bar{b}_2\\ \vdots\\ \bar{b}_n\end{pmatrix} - \begin{bmatrix}S_{22} & \dots & S_{2n}\\ \vdots & \ddots & \vdots\\ S_{n2} & \dots & S_{nn}\end{bmatrix} .\text{diag}\begin{pmatrix}\bar{a}_2\\ \vdots\\ \bar{a}_n\end{pmatrix}\right]^{-1} .\bar{a}_1. \begin{pmatrix}S_{21}\\ \vdots\\ S_{n1}\end{pmatrix} \tag{7}$$

The SOLT calibration is completed with a single forward measurement on a multiport "thru" with previously known SnP values. $(n-1)$ independent measurements, leading to $(n-1)$ transmission terms are performed in one shot. This is the fastest multiport calibration but require a multiport standard presenting few losses on the RF path between port 1 and all other ports.

IV. SOLR METHOD

For the Short-Open-Load-Reciprocal, the "thru" standard SnP values are not necessary. Only an estimation of $\text{Arg}\{S_{thru}(i,j)\}$ for $i \neq j$ is required [7].

By definition, the $[S]$ matrix can be expressed from a and b waves measurements as follow :

$$[S] = \begin{bmatrix}b_{11} & \dots & b_{1n}\\ \vdots & \ddots & \vdots\\ b_{n1} & \dots & b_{nn}\end{bmatrix} \begin{bmatrix}a_{11} & \dots & a_{1n}\\ \vdots & \ddots & \vdots\\ a_{n1} & \dots & a_{nn}\end{bmatrix}^{-1} \tag{8}$$

where a_{ij} and b_{ij} are respectively the incident and reflected waves measured at port i when the RF source is active on port j.

A partially calibrated VNA described on equation (4) will provide partially calibrated S-parameter matrix $[\bar{S}]$ of the multiport device under test:

$$[\bar{S}] = [\bar{b}_{ij}][\bar{a}_{ij}]^{-1} \tag{9}$$

Equation (5) implies the relation between the S-parameters obtained from a full calibrated system ($[S]$) and a partially calibrated system ($[\bar{S}]$).

$$[S] = [\text{diag}(\alpha_1, \dots, \alpha_n)][\bar{S}][\text{diag}(\alpha_1, \dots, \alpha_n)]^{-1} \tag{10}$$

Equation (10) can be written as follow :

$$S_{ij} = \frac{\alpha_i}{\alpha_j}\bar{S}_{ij} \tag{11}$$

The reciprocity assumption on a multiport "unknown thru" ($S_{ij} = S_{ji}$) lead us to the following solution :

$$\alpha_j = \pm\alpha_i.\sqrt{\frac{\bar{S}_{ij}}{\bar{S}_{ji}}} \tag{12}$$

We need an estimation of $\text{Arg}\{S_{ij}\}$ to select the correct solution and we have assumed $\alpha_1 = 1$ in this paper.

A. 2-port SOLR solution

On a 2-port VNA α_2 is given by:

$$\alpha_2 = \pm\sqrt{\frac{\bar{S}_{12}}{\bar{S}_{21}}} \tag{13}$$

A method for automatically determining the proper root selection on a 2-port VNA SOLR algorithm is presented in [8]. The correct solution for α_2 satisfy :

$$\min_{\alpha_2}\left\{\|exp(j.\text{Arg}\{\alpha_2.\bar{S}_{21}\}) - exp(j.\text{Arg}\{S_{21\,est}\})\|\right\} \tag{14}$$

If the “unknown thru” standard is a line or a broadband adapter, we can calculate the estimated phase of S_{21} from a delay as :

$$\text{Arg}\{S_{21\ est}\} = -2\pi.f.\tau_{delay} \tag{15}$$

B. n-port SOLR solution

For multiport SOLR, we start from $\alpha_1 = 1$ and extract every α_j values from equation (12) that is valid on a partially calibrated multiport VNA.

The idea consists on using a multiport “unknowwn thru” that has been previously measured or simulated in S-parameters. Those data are used only to calculate the estimated phase of S_{ij} with $i \neq j$.

The multiport “unknown thru” S-parameters are measured with a partially calibrated system (one connection, but (n^2-n) forward and reverse measurements). Then, the best $(n-1)$ source-receiver configurations for power transfers are identified in order to minimize the measurement noise. This sorting is performed with a Dijkstra’s algorithm.

Dijkstra’s algorithm [9] is the first study to find the shortest path by network traversal. This graph theory algorithm starts from a node and builds the shortest path tree rooted at this node.

The algorithm applied in this paper starts from port $i = 1$ for which $\alpha_i = \alpha_1 = 1$ and spreads the Dijkstra’s tree to calculate α_j associated with the next port (j) as in equation (12).

Figure 1 illustrates a non-ideal 4-port “unknown thru” made with a birectional coupler and an attenuator. The Dijkstra algorithm start from $\alpha_1 = 1$ and calculates α_2. Then α_3 is obtained from α_1 and finally α_4 is identified from α_3.

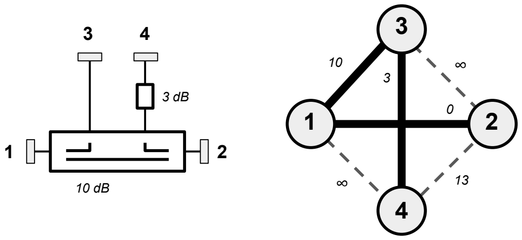

Fig. 1. Example of multiport unknown thru standard and its associated RF path selected by the Dijkstra algorithm. Path weight between nodes i and j are the estimated losses of the transmission expressed in dB : $w_{ij} = 20.log_{10}(|S_{ij}|)$. Finally, the algorithm will select S_{12}, S_{13} and S_{34} measurements to respectively calculate error terms $\alpha 2$, α_3 and α_4.

In our NVNA calibration study, nodes are RF ports and paths are weighted with the power losses of our multiport standard. The Dijkstra algorithm finds the best spanning tree among the n^{n-2} (Cayley’s formula) possibilities on a n-port system.

V. EXPERIMENT

The proposed calibration methods have been processed by a 4-port Rohde & Schwarz ZVA 24 with four 3.5mm male type port . A frequency range of 2 to 18 GHz, an IF bandwidth of 10 kHz and 201 points were defined in the VNA.

A. Physical “unknown thru” standards

The three “unknown thru” standards considered in this studies are illustrated on Figure 2.

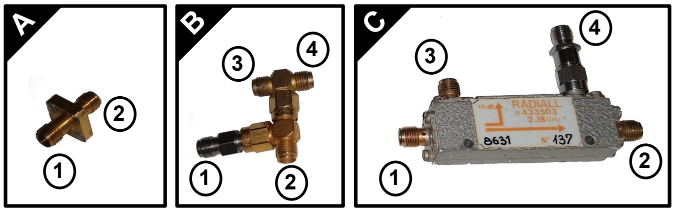

Fig. 2. The three “unknown thru” on use in this study.

- Standard A is a 3.5mm female-female adapter with a time goup delay $\tau = 58ps$. This two port standard will be used to calibrate the 4-port VNA with a traditional multiport SOLR method (the “unknown thru” is connected sequentially to port 2, 3 and 4). This calibration will be our reference when we will compare our new SOLT and SOLR methods.
- Standard B is a 4-port unknown thru made with 2 SMA tee adapters. On this circuit, losses are lower than 10 dB over the 2-18 GHz range but not flat. Matching is not good.
- Standard C is made with a bidirectional coupler with an aditional attenuator on its port 4 as illustrated on Figure 1. This 4-port unknown thru is not ideal because of the isolation of the coupler (i.e. between ports 1 and 4) but matching is great and transfer response flat over the 2-18 GHz bandwidth.

B. Multiport calibration results

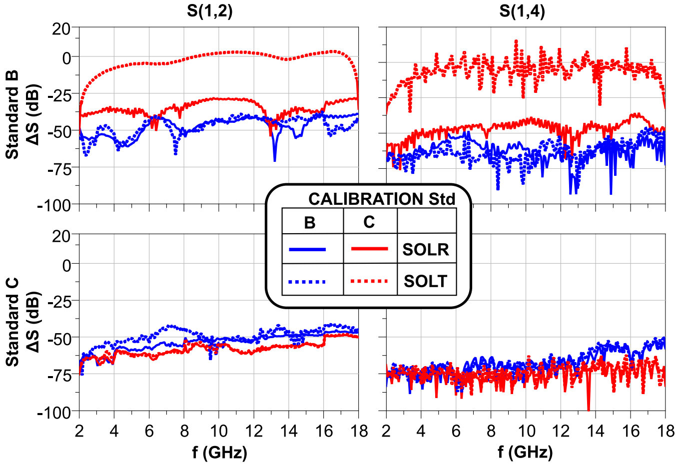

Fig. 3. Comparison results of the new multiport SOLT and SOLR methods. ΔS is the vectorial transmission tracking error term difference between the new calibration methods and the conventional SOLR method using the 2-port standard ‘A’. The new calibration methods displayed here are multiport SOLT and SOLR using standards ‘B’ or ‘C’.

Figure 3 illustrates the measurement offset between the traditional multiport SOLR method (applied with standard A) the new multiport SOLT and SOLR methods described in this paper applied with standards B and C.

ΔS is the magnitude (in dB) of the vectorial difference between the VNA calibrated with a traditionnal calibration and a new calibration on a measured S-parameters device. Differences with tne new SOLT and SOLR are plotted with dots and continuous lines respectively. A new calibration performed with standard B is plotted in blue, and with standard C in red. The measured devices are standard B (top) and standard C (bottom).

The multiport SOLT is the fastest method because only one forward measurement from port 1 is needed. But the quality of the calibration will depend on the losses presented by the multiport thru : here the VNA can not be calibrated with the bidirectional coupler because of its isolation between ports 1 and 4. The multiport SOLR is more robust. Its large amount of measurement associated with the Dijkstra sorting ensure the best possible calibration independently of the “unknown thru” standard. For NVNA measurements, both multiport SOLT and SOLR present acceptable offset compared to the traditional SOLR multiport method but the multiport SOLT method requires a wise selection on the multiport “thru” standard.

VI. CONVERSION TO MULTIPORT 12-TERM ERROR MODEL

Multiport calibration is usualy performed over a 12-term error model given by a GSOLT model [10]. Conversion between 8-terms and 12-terms multiport calibration is given in [11] for VNA formalism. The 8-term multiport SOLT and SOLR methods, presented in this paper, are defined in a NVNA formalism. NVNA error-terms conversion into a conventional GSOLT model is presented here. Figure 4 shows a 12-term error model diagram (forward mode) when the source is activated on port i and signals are measured on port i and j. 12-term error model consists on 6 error terms: three releflection terms and three transmission terms (one of them is the “optional” isolation). This paragraph presents the equations to convert multiport NVNA error terms in GSOLT VNA error terms.

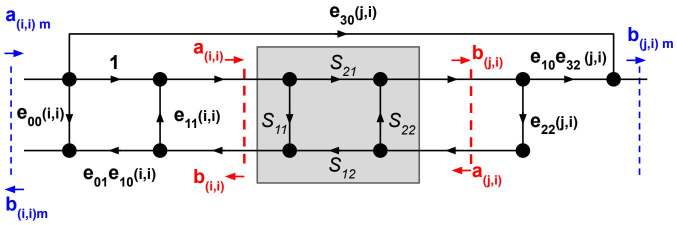

Fig. 4. General 12-term error model of a multiport VNA. This figure illustrates the flow graph and the 6 error terms related to a transmission between port i and port j when the source is active on port i. Most VNA firmwares store error terms in this format. Uncalibrated waves are displayed in blue and calibrated ones in red.

Multiport reflection terms are expressed with 3 diagonal matrices :

- Directivity

$$[e_{00}] = \operatorname{diag}\left(-\frac{\gamma_i}{\delta_i}\right) \tag{16}$$

- Reflection Tracking

$$[e_{11}] = \operatorname{diag}\left(\frac{\beta_i}{\delta_i}\right) \tag{17}$$

- Source Match

$$[e_{01}e_{10}] = \operatorname{diag}\left(\frac{\alpha_i\delta_i - \beta_i\gamma_i}{(\delta_i)^2}\right) \tag{18}$$

Multiport transmission terms are defined with 3 square matrices with a zero diagonal :

- Load Match

$$[e_{22}(j,i)] = \begin{cases} \frac{\bar{a}(j,i)}{\bar{b}(j,i)} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} \tag{19}$$

- Transmission Tracking

$$[e_{10}e_{32}(j,i)] = \begin{cases} \frac{\alpha_i\delta_i - \beta_i\gamma_i}{\delta_i\delta_j} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} \tag{20}$$

- Crosstalk (Isolation)

$$[e_{30}(j,i)] = \begin{cases} \frac{b_{j,i\;m}}{a_{i,i\;m}} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} \tag{21}$$

VII. CONCLUSION

This paper studies on the theory and algorithms of SOLT and SOLR 8-error term models for multiport NVNA. It has been demonstrated that this new multiport “thru” and “unknown thru” calibration greatly simplifies the calibration steps necessary to measure multiport devices. Theses techniques speed up multiport measurements and are particulary welcome in the framework of MIMO system large signal characterizations as multiple-input power amplifiers or beamformers dedicated to phased array antennas.

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